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Sequential detection of a weak target in a hostile ocean environment

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Abstract—When the underlying physical phenomenology (medium, sediment, bottom, etc.) is space-time varying along with corresponding nonstationary statistics characterizing noise and uncertainties, then sequential methods must be applied to capture the underlying processes. Sequential detection and estimation techniques offer distinct advantages over batch methods. A reasonable signal processing approach to solve this class of problem is to employ adaptive or parametrically adaptive signal models and noise to capture these phenomena. In this paper, we develop a sequential approach to solve the signal detection problem in a nonstationary environment.

I. INTRODUCTION

The advent of quiet nuclear submarines and the current proliferation of even quieter diesel powered vessels have necessitated the need for more sophisticated signal processing techniques for target detection. Improvements in processor design can be achieved by developing approaches that incorporate knowledge of the surrounding ocean environment and noise in the form of mathematical models into their processing schemes [1-4]. One of the major problems with model-based signal processing schemes is assuring that the model incorporated in the algorithm is adequate for the proposed application and that it can faithfully represent the on-going phenomenology. Therefore, it is necessary, as part of the model-based processor (MBP) design procedure, to estimate/update the model parameters jointly (adaptively) while performing the required processing [5,6]. The incorporation of a parametrically adaptive, on-line, MBP can offer a dramatic detection improvement in a tactical passive or active sonar-type system especially when a rapid environmental assessment and detection is required. In this paper, we discuss the development of a processor capable of adapting to the ever-changing ocean environment thereby providing the required signal enhancement for detection.

With this motivation in mind, we investigate the development of a sequential detector, that is, a monitor that incorporates an initial mathematical representation of the weak target model into its framework and adapts, on-line, its parameters as the ocean and target change. In this paper we use an adaptive state-space scheme [7] and apply it to the detection and monitoring problem. This can be accomplished by constructing an adaptive MBP that allows continuous updating of the model parameters and is easily implemented by augmenting them into the current state vector [7-9]. Currently, techniques that adjust model parameters to adapt to the changing environment are termed *environmentally adaptive*.

We start by developing a solution to the general sequential detection problem. Here we assume generic signal models, uncertainty and noise. It is shown that the solution can be obtained using the Wald sequential theoretic framework leading to a recursive log-likelihood solution under the Neyman-Pearson criterion [10]. Once the basic detector is developed, structurally, we extend the results to solve the composite detection problem with unknown parameters leading to the generalized (sequential) log-likelihood solution. Here we incorporate both the log-likelihood decision function and adaptive parameter estimation into the sequential framework leading to a joint detection/signal enhancement solution.

Special cases of the generic signal model are then investigated (e.g. far-field sources, unknown frequencies, bearings, etc.) and cast into the sequential framework to develop the underlying theory required for each. It is shown that this formulation of the detection problem for nonstationary ocean acoustic signals leads to a general approach capable of solving a large number of detection/enhancement problems.

II. SEQUENTIAL DETECTION THOERY

In this section we develop the sequential detection scheme for nonstationary processes. We start with the batch solution and then develop the sequential approach. First we assume that we have a set of vector measurements defined by $P_t := \{\mathbf{p}(0), \mathbf{p}(1), \cdots, \mathbf{p}(t)\}$, $\mathbf{p} \in \mathbb{C}^{L \times 1}$ and we would like to

develop a processor based on the usual binary decision problem [10]:

$$H_o : \ \mathbf{p}(t) = \mathbf{v}(t) \qquad [\text{ Broadband Noise}]$$
$$H_1 : \ \mathbf{p}(t) = \mathbf{s}(t) + \mathbf{v}(t) \qquad [\text{Signal, Broadband Noise }] \qquad (1)$$

where the null hypothesis is noise and the alternate is the signal and noise case. We choose the Neyman-Pearson criterion to develop our detector [10].

Recall that the Neyman-Pearson theorem states that a detector is optimal if it maximizes the probability of detection, Pr_{DET} (or minimizes the miss probability) for any false alarm rate less than a pre-specified value, say Pr_{FA}^*. The theorem follows directly from a constrained optimization problem formulation using Lagrange multipliers (see [10,11] for details), which yields the solution based on the ratio of likelihood's, $\text{Pr}(\text{P}_t \mid H_i); \ i = 0,1$ (for the binary case). That is, to maximize Pr_{DET} for a fixed value of $\text{Pr}_{\text{FA}} = \text{Pr}_{\text{FA}}^*$, we have the *likelihood ratio*, $L(t)$ or equivalently sufficient statistic defined by the joint density functions

$$L(t) := L(\text{P}_t) = \frac{\text{Pr}(\text{P}_t \mid H_1)}{\text{Pr}(\text{P}_t \mid H_o)} \quad \begin{matrix} > \text{Accept } H_1 \\ \\ < \text{Accept } H_o \end{matrix} \quad \text{T}, \qquad (2)$$

for P_t and $\text{Pr}(\cdot)$ are the respective joint probabilities under each hypothesis. Here the threshold T is found from the false alarm density for the pre-specified value

$$\text{Pr}_{\text{FA}} = \int_{\text{T}}^{\infty} \text{Pr}(\text{P}_t \mid H_o) dP = \text{Pr}_{\text{FA}}^*. \qquad (3)$$

Thus, based on the Neyman-Pearson criterion, the optimal detector can be found for "batch" mode as

$$L(t) = \frac{\text{Pr}(\text{P}_t \mid H_1)}{\text{Pr}(\text{P}_t \mid H_o)} = \frac{\text{Pr}(\mathbf{p}(t),\cdots,\mathbf{p}(0) \mid H_1)}{\text{Pr}(\mathbf{p}(t),\cdots,\mathbf{p}(0) \mid H_o)}. \qquad (4)$$

It follows from the chain rule of probability theory [10] that

$$\begin{aligned} \text{Pr}(\text{P}_t \mid H_i) &= \prod_{k=0}^{t} \text{Pr}(\mathbf{p}(t-k) \mid \text{P}_{t-k-1};H_i) \\ &= \text{Pr}(\mathbf{p}(t) \mid \text{P}_{t-1};H_i) \times \text{Pr}(\mathbf{p}(t-1) \mid \text{P}_{t-2};H_i) \times \cdots \times \\ &\quad \text{Pr}(\mathbf{p}(1) \mid \mathbf{p}(0);H_i) \times \text{Pr}(\mathbf{p}(0);H_i) \end{aligned}$$
$$(5)$$

which can be expressed succinctly using Bayes' rule as

$$\text{Pr}(\text{P}_t \mid H_i) = \text{Pr}(\mathbf{p}(t),\text{P}_{t-1} \mid H_i) = \text{Pr}(\mathbf{p}(t) \mid \text{P}_{t-1};H_i)\text{Pr}(\text{P}_{t-1} \mid H_i)$$
$$(6)$$

Substituting these expressions into Eq. (2), we obtain

$$L(t) = \left[\frac{\text{Pr}(\text{P}_{t-1} \mid H_1)}{\text{Pr}(\text{P}_{t-1} \mid H_o)} \right] \frac{\text{Pr}(\mathbf{p}(t) \mid \text{P}_{t-1};H_1)}{\text{Pr}(\mathbf{p}(t) \mid \text{P}_{t-1};H_o)} = L(t-1) \frac{\text{Pr}(\mathbf{p}(t) \mid \text{P}_{t-1};H_1)}{\text{Pr}(\mathbf{p}(t) \mid \text{P}_{t-1};H_o)}$$
$$(7)$$

which is precisely the sequential form of the likelihood ratio. It is also clear that not just the likelihood function can be used but any monotonic function of the likelihood can also be used as well [10,11]. Taking natural logarithms of both sides of the equation, and defining $\Lambda(t) := \ln L(t)$, we obtain the *sequential log-likelihood ratio* as

$$\Lambda(t) = \Lambda(t-1) + \ln \text{Pr}(\mathbf{p}(t) \mid \text{P}_{t-1};H_1) - \ln \text{Pr}(\mathbf{p}(t) \mid \text{P}_{t-1};H_o). \quad (8)$$

To construct the sequential probability ratio test (SPRT) or equivalently the *sequential likelihood ratio test* (SLRT) based on the Neyman-Pearson criterion, we must define two time-varying thresholds $\text{T}_o(t)$ and $\text{T}_1(t)$. If the likelihood ratio at time t is greater than threshold $\text{T}_1(t)$, we accept hypothesis H_1. If it is less than $\text{T}_o(t)$, we accept H_o, but if its value lies between the thresholds, we *continue* to take another sample. The sequential test differs from the fixed sample size "batch" test and is capable of handling non-stationary statistics as we will show in a subsequent section.

To implement the SLRT at the tth stage (see Eq. (7) or (8)), we must know the prior probabilities or equivalently the prior likelihood ratio, $L(t-1)$ (or $\Lambda(t-1)$) containing all of the past information and the corresponding stage conditional probabilities $\text{Pr}(\mathbf{p}(t) \mid \text{P}_{t-1};H_i)$ to sequentially update the likelihood. The tth stage thresholds are construct following Neyman-Pearson theory in terms of the detection and false alarm probabilities as

$$\text{T}_o(t) = \frac{\text{Pr}_{\text{DET}}}{\text{Pr}_{\text{FA}}} = \frac{1 - \text{Pr}_{\text{MISS}}}{\text{Pr}_{\text{FA}}}, \quad \text{and} \quad \text{T}_1(t) = \frac{1 - \text{Pr}_{\text{DET}}}{1 - \text{Pr}_{\text{FA}}} = \frac{\text{Pr}_{\text{MISS}}}{1 - \text{Pr}_{\text{FA}}}$$
$$(9)$$

In the case of the log-likelihood, the thresholds must be transformed as well, that is,

$$\begin{array}{ll} \Lambda(t) \geq \ln \text{T}_1(t) & [\text{Accept } H_1] \\ \ln \text{T}_o(t) < \Lambda(t) < \ln \text{T}_1(t) & [\text{Continue}] \qquad (10) \\ \Lambda(t) \leq \ln \text{T}_o(t) & [\text{Accept } H_o] \end{array}$$

This completes the fundamental concepts for the construction of the sequential detection approach; however, before we close this section, we must consider the case of incorporating unknown parameters into the binary detection problem, which will lead to the generalized likelihood tests [11].

If the conditional densities of the underlying problem contain

a set of unknown parameters defined by the vector $\boldsymbol{\Theta}$, then two cases must be considered for sequential detection. In the first case, $\boldsymbol{\Theta}$ is considered to be *random* and therefore *joint* conditional probabilities, $\Pr\left(P_t, \boldsymbol{\Theta} \mid H_i\right)$ must be incorporated and the likelihood is now computed in terms of the marginals

$$L(t) = \frac{\Pr\left(P_{t-1} \mid H_1\right)}{\Pr\left(P_{t-1} \mid H_o\right)} = \frac{\int \Pr\left(P_{t-1}, \boldsymbol{\Theta} \mid H_1\right) d\Theta}{\int \Pr\left(P_{t-1}, \boldsymbol{\Theta} \mid H_o\right) d\Theta}$$
$$= \frac{\int \Pr\left(P_{t-1} \mid \boldsymbol{\Theta}; H_1\right) \Pr\left(\boldsymbol{\Theta} \mid H_1\right) d\Theta}{\int \Pr\left(P_{t-1} \mid \boldsymbol{\Theta}; H_o\right) \Pr\left(\boldsymbol{\Theta} \mid H_o\right) d\Theta}$$

(11)

by integrating over $\boldsymbol{\Theta}$, the likelihood ratio is determined and in a sense, the composite problem is reduced to simple hypothesis testing.

In the second case, $\boldsymbol{\Theta}$ is considered to be deterministic but *unknown* and the above approach cannot be applied. Another approach is to estimate the unknown parameter vector $\hat{\boldsymbol{\Theta}} \to \boldsymbol{\Theta}$ under each hypothesis and proceed with the simple testing. A *maximum likelihood estimate,* $\hat{\boldsymbol{\Theta}}_{\text{ML}}$, can be used to create the *generalized likelihood ratio test* (GLRT) defined by

$$L_{\boldsymbol{\Theta}}(t) := L_{\boldsymbol{\Theta}}(P_t) = \frac{\max\limits_{\boldsymbol{\Theta}_1} \Pr\left(P_t \mid \boldsymbol{\Theta}_1; H_1\right)}{\max\limits_{\boldsymbol{\Theta}_o} \Pr\left(P_t \mid \boldsymbol{\Theta}_o; H_o\right)}$$

(12)

This is the approach we use in this paper for our adaptive processor; therefore, the GLRT is defined by

$$L_{\boldsymbol{\Theta}}(t) = \frac{\Pr\left(P_t \mid \boldsymbol{\Theta}_1, H_1\right)}{\Pr\left(P_t \mid \boldsymbol{\Theta}_o, H_o\right)} = L_{\boldsymbol{\Theta}}(t-1) \frac{\Pr\left(\mathbf{p}(t) \mid P_{t-1}; \boldsymbol{\Theta}_1, H_1\right)}{\Pr\left(\mathbf{p}(t) \mid P_{t-1}; \boldsymbol{\Theta}_o, H_o\right)}$$

(13)

or in logarithmic form as

$$\Lambda_{\boldsymbol{\Theta}}(t) = \Lambda_{\boldsymbol{\Theta}}(t-1) + \ln \Pr(\mathbf{p}(t) \mid P_{t-1}; \boldsymbol{\Theta}_1, H_1) - \ln \Pr(\mathbf{p}(t) \mid P_{t-1}; \boldsymbol{\Theta}_o, H_o)$$

(14)

with the corresponding thresholds in terms of the parameters

$$T_o\left(t; \boldsymbol{\Theta}\right) := \frac{1 - \Pr_{\text{MISS}}\left(\boldsymbol{\Theta}\right)}{\Pr_{\text{FA}}\left(\boldsymbol{\Theta}\right)}, \quad \text{and} \quad T_1\left(t; \boldsymbol{\Theta}\right) := \frac{\Pr_{\text{MISS}}\left(\boldsymbol{\Theta}\right)}{1 - \Pr_{\text{FA}}\left(\boldsymbol{\Theta}\right)}$$

for

$$\Pr_{\text{FA}}\left(\boldsymbol{\Theta}\right) = \int_{T_o}^{\infty} \Pr\left(P_t \mid \boldsymbol{\Theta}_o, H_o\right) dP; \Pr_{\text{MISS}}\left(\boldsymbol{\Theta}\right) = \int_{T_o}^{\infty} \Pr\left(P_t \mid \boldsymbol{\Theta}_1, H_1\right) dP$$

(15)

This completes the background theory, next we apply it to our ocean acoustic problem.

III. JOINT SEQUENTIAL DETECTION/SIGNAL ENHANCEMENT

In this section we develop a solution to the joint detection /signal enhancement problem for a towed array in the ocean. We start with the noisy *pressure-field* measurement given by

$$p(r_\ell; t) = s(r_\ell; t) + v(r_\ell; t), \tag{16}$$

where p is the measured pressure-field at the ℓth-hydrophone located at spatial location, r_ℓ, and at time t; s is the target or source signal to be detected and v is the broadband noise component present in the ocean environment. We can simplify this notation by expanding over the horizontal array of L-elements, that is,

$$\mathbf{p}(t) = \mathbf{s}(t) + \mathbf{v}(t), \tag{17}$$

with $\mathbf{p}, \mathbf{s}, \mathbf{v} \in \mathbb{C}^{L \times 1}$. We decompose this representation further by developing the component signal and noise models. We assume that the *signal* can be characterized by a *weak* target in the far-field of the array given by

$$s(t) = \alpha_o e^{i(\omega_o t - \mathbf{k}_o \cdot \mathbf{r})} = \alpha_o e^{i(\omega_o t - k_o \sin\theta_o(r_o + vt))}, \tag{18}$$

for the target parameters: α_o, ω_o, k_o, θ_o, r_o that are the respective amplitude, temporal frequency, wavenumber, bearing angle and initial sensor location. Since the array is being towed, we include the tow speed, v as well. We can simplify this model by defining the following terms,

$$s(t) = \alpha_o(t) e^{-i\beta_o(t)\sin\theta_o}, \tag{19}$$

for $\alpha_o(t) := \alpha_o e^{i\omega_o t}$ and $\beta_o(t) := k_o\left(r_o + vt\right)$. The broadband measurement noise is modeled as zero-mean, white gaussian, $\mathbf{v} \sim N\left(0, R_{vv}(t)\right)$. Note that we are not restricting the statistics to be stationary, so we can accommodate the nonstationarities (transients, etc.) that occur naturally in the ocean environment.

For the joint detection/signal enhancement problem under the null hypothesis, we have that $\Pr(\mathbf{p}(t) \mid P_{t-1}; H_o)$ is a conditionally gaussian distribution, since $\mathbf{v}$ is white. Therefore the conditional density is given by $\Pr\left(\mathbf{p}(t) \mid P_{t-1}; H_o\right) \sim N\left(0, R_{vv}(t)\right)$. For various assumptions on the structure of the signal model the density changes. We start with the simplest signal model and proceed.

Case I: *Known* signal $\mathbf{s}(t)$ with $\Pr(\mathbf{p}(t) \mid P_{t-1}; H_1) \sim N\left(\hat{\mathbf{p}}(t \mid t-1), R_{e_p e_p}(t)\right)$: For this case we have the following conditional statistics

$$\hat{\mathbf{p}}(t\,|\,t-1) = E\{\mathbf{s}(t)+\mathbf{v}(t)\,|\,P_{t-1}\} = \mathbf{s}(t)$$
$$\mathbf{e}_{\mathrm{p}}(t) = \mathbf{p}(t)-\hat{\mathbf{p}}(t\,|\,t-1) = \mathbf{s}(t)+\mathbf{v}(t)-\mathbf{s}(t) = \mathbf{v}(t) \quad (20)$$
$$R_{\mathbf{e}_{\mathrm{p}}\mathbf{e}_{\mathrm{p}}}(t) = R_{\mathbf{vv}}(t)$$

and the corresponding log-likelihood is therefore

$$\Lambda(t) = \Lambda(t-1)+\Delta K + \frac{1}{2}\big(\mathbf{p}(t)-\mathbf{s}(t)\big)' R_{vv}^{-1}(t)\big(\mathbf{p}(t)-\mathbf{s}(t)\big)$$
$$-\frac{1}{2}\mathbf{p}'(t)R_{vv}^{-1}(t)\mathbf{p}(t) \qquad (21)$$

Simplifying terms Eq. (21) becomes

$$\Lambda(t) = \Lambda(t-1)+\Delta K + \mathbf{p}'(t)R_{vv}^{-1}(t)\mathbf{s}(t)+\mathbf{s}'(t)R_{e_{p}e_{p}}^{-1}(t)\mathbf{s}(t) \quad (22)$$

where $\Delta K := K_1 - K_o = \frac{1}{2}\Big(\ln\big|R_{vv}(t)\big|-\ln\big|R_{e_{p}e_{p}}(t)\big|\Big)$. Moving
the terms that are not a function of the measurement into the
threshold gives the classical *matched-filter* result [10,11].

Case II: *Unknown* $\mathbf{s}(t;\Theta)$ for $\mathrm{Pr}\big(\mathbf{p}(t;\Theta)\,|\,\mathrm{P}_{t\text{-}1};\Theta,H_1\big) \sim \mathrm{N}\big(\hat{\mathbf{p}}(t\,|\,t-1;\Theta),R_{e_{p}e_{p}}(t;\Theta)\big)$

In this case the signal structure is known from Eq. (18), but
not the parameters. The conditional statistics are therefore

$$\hat{\mathbf{p}}(t\,|\,t-1;\Theta) = E\{\mathbf{s}(t)+\mathbf{v}(t)\,|\,P_{t-1};\Theta\} = \hat{\mathbf{s}}(t;\Theta)$$
$$\mathbf{e}_{\mathrm{p}}(t;\Theta) = \mathbf{p}(t)-\hat{\mathbf{p}}(t\,|\,t-1;\Theta) = \big(\mathbf{s}(t)-\hat{\mathbf{s}}(t;\Theta)\big)+\mathbf{v}(t) = \tilde{\mathbf{s}}(t;\Theta)+\mathbf{v}(t)$$
$$R_{\mathbf{e}_{\mathrm{p}}\mathbf{e}_{\mathrm{p}}}(t;\Theta) = R_{\tilde{s}\tilde{s}}(t;\Theta)+R_{vv}(t)$$
$$(23)$$

Since $\mathbf{s}(t)$ is assumed to be a far-field source, we have that at
the ℓth-sensor, $s_\ell(t) = \alpha_\ell(t)e^{-i\beta_\ell(t)\sin\theta}$. Now expanding
over the L-sensor array, we obtain the signal vector

$$\mathbf{s}(t) = \begin{bmatrix} \alpha(t)e^{-ik\beta_1(t)\sin\theta} \\ \vdots \\ \alpha(t)e^{-ik\beta_L(t)\sin\theta} \end{bmatrix} = \begin{bmatrix} \alpha e^{i\omega t}e^{-ik(x_1+vt)\sin\theta} \\ \vdots \\ \alpha e^{i\omega t}e^{-ik(x_L+vt)\sin\theta} \end{bmatrix}, (24)$$

Signal enhancement can be achieved by defining the signal
vector in terms of its unknown parameters, $\mathbf{s}(t;\Theta)$, for (a
single target), $\Theta := [\alpha\,|\,\omega\,|\,\theta]'$. In this case we assume that
the unknown parameters in the signal model, Θ, are
characterized by a piecewise constant representation ($\dot{\Theta}=\mathbf{0}$)
This parameter vector is the state vector and therefore the
underlying discrete Gauss-Markov model for this problem is

$$\Theta(t) = \Theta(t-1)+\Delta t\,\mathbf{w}_\Theta(t-1)$$
$$\mathbf{p}(t) = \mathbf{c}[t;\Theta]+\mathbf{v}(t) = \mathbf{s}(t;\Theta)+\mathbf{v}(t) \qquad (25)$$

where $\mathbf{w}_\Theta \sim \mathrm{N}(0,R_{\mathbf{w}_\Theta\mathbf{w}_\Theta})$. Thus, the basic detection/signal
enhancement problem can now be stated as

> GIVEN a set of discrete noisy pressure-field
> measurements, $\{\mathbf{p}(t)\}$, $t=1,2,\cdots,N_t$ and the Gauss-
> Markov model of Eq. (25), FIND the optimal (Neyman-
> Pearson) solution for joint detection/enhancement. That
> is, find the log-likelihood $\Lambda_\Theta(t)$ and the best (minimum
> variance) estimate of the signal , $\hat{\mathbf{s}}(t;\Theta)$ or equivalently
> the parameter vector $\hat{\Theta}(t\,|\,t)$.

Using the GLRT of Eq. (12) to sequentially perform the
detection, we must estimate the parameters on-line as well. It
is well known from recursive estimation theory that a MBP
can be used to sequentially estimate these parameters based
on this Gauss-Markov formulation. We have a linear state-
space, but unfortunately a nonlinear measurement system
requiring a nonlinear processor. This problem can be solved
by a parametrically adaptive MBP using the recursive
extended Kalman filter (EKF) given in Table I to provide the
maximum likelihood parameter estimates (gaussian case) for
the optimal detector [5,6,9].

TABLE I
ADAPTIVE MODEL-BASED SIGNAL ENHANCER

PREDICTOR
$\hat{\Theta}(t\,|\,t-1) = \hat{\Theta}(t-1\,|\,t-1)$
$\tilde{P}(t\,|\,t-1) = \tilde{P}(t-1\,|\,t-1)+R_{\mathbf{w}_\Theta\mathbf{w}_\Theta}(t-1)$
INNOVATIONS
$\mathbf{e}_{\mathrm{p}}(t) = \mathbf{p}(t)\text{-}\hat{\mathbf{p}}(t\,|\,t-1)$
$\hat{\mathbf{p}}(t\,|\,t-1) = \mathbf{c}\big[t;\hat{\Theta}\big] = \mathbf{s}\big(t;\hat{\Theta}(t\,|\,t-1)\big)$
$R_{\mathbf{e}_{\mathrm{p}}\mathbf{e}_{\mathrm{p}}}(t) = \bar{\mathbf{C}}_\Theta\big[\hat{\Theta}\big]\tilde{P}(t\,|\,t-1)\bar{\mathbf{C}}'_\Theta\big[\hat{\Theta}\big]+R_{vv}(t)$
GAIN
$\mathbf{K}_\Theta(t) = \tilde{P}(t\,|\,t-1)\bar{\mathbf{C}}'_\Theta\big[\hat{\Theta}\big]R_{\mathbf{e}_{\mathrm{p}}\mathbf{e}_{\mathrm{p}}}^{-1}(t)$
CORRECTOR
$\hat{\Theta}(t\,|\,t) = \hat{\Theta}(t\,|\,t-1)+\mathbf{K}_\Theta(t)\mathbf{e}_{\mathrm{p}}(t)$
$\tilde{P}(t\,|\,t) = \big(\mathbf{I}\text{-}\mathbf{K}_\Theta(t)\bar{\mathbf{C}}_\Theta\big[\hat{\Theta}\big]\big)\tilde{P}(t\,|\,t-1)$
ENHANCER
$\hat{\mathbf{s}}(t;\Theta) = \begin{bmatrix} \hat{\Theta}_1(t\,|\,t)e^{i\hat{\Theta}_2(t|t)t}e^{-ik(x_1+vt)\sin\hat{\Theta}_3(t|t)} \\ \vdots \\ \hat{\Theta}_1(t\,|\,t)e^{i\hat{\Theta}_2(t|t)t}e^{-ik(x_L+vt)\sin\hat{\Theta}_3(t|t)} \end{bmatrix}$
$\bar{\mathbf{A}}_\Theta\big[\hat{\Theta}\big] := \dfrac{\partial\mathbf{A}_\Theta}{\partial\Theta}\bigg|_{\Theta=\hat{\Theta}}$; $\bar{\mathbf{C}}_\Theta\big[\hat{\Theta}\big] := \dfrac{\partial\mathbf{c}[\Theta]}{\partial\Theta}\bigg|_{\Theta=\hat{\Theta}}$

Using the EKF algorithm it is necessary to provide the jacobians for implementation, that is,

$$\frac{\partial A_\Theta[\Theta]}{\partial \Theta} = \mathrm{I}, \quad \frac{\partial c[\Theta]}{\partial \theta} = i\alpha(t)\beta_\ell(t)\cos\theta e^{i\beta_\ell(t)\sin\theta}$$

$$\frac{\partial c[\Theta]}{\partial \omega} = it\alpha(t)e^{i\beta_\ell(t)\sin\theta}, \quad \frac{\partial c[\Theta]}{\partial \alpha} = e^{i(\omega t - \beta_\ell(t)\sin\theta)} \quad (26)$$

$$\ell = 1, \cdots, L$$

completing the development of the parametrically adaptive solution.

Using the same approach as Case I, we obtain the optimal decision function

$$\Lambda_\Theta(t) = \Lambda_\Theta(t-1) + \Delta K + \mathbf{e}'_\mathbf{p}(t;\Theta)R^{-1}_{e_p e_p}(t;\Theta)\mathbf{e}_\mathbf{p}(t;\Theta) - \mathbf{p}'(t)R^{-1}_{vv}(t)\mathbf{p}(t) \quad (27)$$

The innovations and its covariance are given by the following (see Table I for details)

$$\mathbf{e}_\mathrm{p}(t;\Theta) \quad = \mathbf{p}(t)\text{-}\hat{\mathbf{p}}(t\,|\,t-1)$$

$$\hat{\mathbf{p}}(t\,|\,t-1;\Theta) = \mathbf{c}\big[t;\hat{\Theta}\big] = \mathbf{s}\big(t;\hat{\Theta}(t\,|\,t-1)\big) \qquad . \quad (28)$$

$$R_{\mathbf{e}_\mathrm{p}\mathbf{e}_\mathrm{p}}(t;\Theta) \quad = \bar{C}_\Theta\big[\hat{\Theta}\big]\tilde{P}(t\,|\,t-1)\bar{C}'_\Theta\big[\hat{\Theta}\big] + R_{vv}(t)$$

So we see that the effect of the unknown signal model is to complicate the log-likelihood decision function by including the additional information. However, assuming that this knowledge is accurate, the correct decision can be accomplished at a much lower SNR---which is the main objective of this MBP scheme.

Thus, the joint parametrically adaptive processor is capable of not only providing a the optimal detection solution $\big(\Lambda_\Theta(t)\big)$ in a sequential manner with nonstationary statistics incorporated, but also capable of adaptively estimating the weak far-field target signal for optimal signal enhancement $\big(\hat{\mathbf{s}}(t;\Theta) \to \mathbf{s}(t)\big)$. Thus, the sequential approach does in fact provide a solution to the joint detection/enhancement problem. Next we summarize our results and discuss future efforts.

III. SUMMARY

In this paper we have developed a solution to the joint detection/signal enhancement problem using a model-based approach [5]. Starting with the optimal (Neyman-Pearson) detector solution, we developed the corresponding model-based solution demonstrating their equivalence for the case where the signal is known a priori. Next we developed the solution to the joint problem with the signal unknown, but parameterized as a far-field source. The solution to this problem lead to the parametrically adaptive model-based processor implemented with the nonlinear extended Kalman filter (EKF) algorithm. It was shown how to design the processor for this problem.

Future efforts will be aimed at applying this technique to both simulated and measured hydrophone data. We plan to use the discrete implementation of the factorized EKF available in MATLAB [12] with the toolbox SSPACK_PC [13].

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